Crystal morphology and surface structures of orthorhombic MgSiO$_3$ perovskite

Abstract Orthorhombic MgSiO$_3$ perovskite is thought to be the most abundant mineral in the mantle of the Earth. Its bulk properties have been widely studied, but many geophysical and rheological processes are also likely to depend upon its surface and grain boundary properties. As a first step towards modelling these geophysical properties, we present here an investigation of the structures and energetics of the surfaces of MgSiO$_3$-perovskite, employing both shell-model atomistic effective-potential simulations, and density-functional-theory (DFT) calculations. Our shell-model calculations predict the {001} surfaces to be the energetically most stable surfaces: the calculated value of the surface energy being 2.2 J/m$^2$ for the MgO-terminated surface, which is favoured over the SiO$_2$-terminated surface (2.7 J/m$^2$). Also for the polar surfaces {111}, {101} and {011} the MgO-terminated surfaces are energetically more stable than the Si-terminated surfaces. In addition we report the predicted morphology of the MgSiO$_3$ perovskite structure, which is dominated by the energetically most stable {001} and {110} surfaces, and which appears to agree well with the shape of grown single crystals.

Keywords Perovskite · Morphology · Surface structures · Grain boundary · Mantle · Modelling

Introduction

Perovskite structured MgSiO$_3$ is believed to comprise 70% of the lower mantle, and it is thus the most abundant mineral in the Earth. Numerous experimental investigations on the MgSiO$_3$ structure have been reported, the majority of which have focussed on its bulk properties (e.g. Fiquet et al. 2000; Fiquet et al. 1998; D. Yamazaki et al. 2000; Bolfan-Casanova et al. 2002; Andrault et al. 2001; Chen et al. 2002; Fei and Wang 2000). However, experimental studies at representative mantle pressures and temperatures are difficult, and so theoretical methods have also been of great value in predicting some of the properties of MgSiO$_3$ under the pressure and temperature conditions to be found in the mantle (e.g. Oganov et al. 2000; Oganov et al. 2001; Parlinski and Kawazoe 2000; Karki et al. 2001; Karki et al. 2000; Stixrude et al. 1998; Stixrude and Cohen 1993; Marton and Cohen 2002; Matsui 2000). Despite these high quality, first principles and inter-atomic potential calculations on the bulk behaviour of MgSiO$_3$ perovskite, no detailed modelling of its surfaces has so far been reported. The aim of the current investigation is to determine the geometrical structures and energies of the surfaces of MgSiO$_3$ perovskite. Surface calculations are often highly demanding of computer resources, so we have used both high level ab initio calculations, as well as the more computationally efficient inter-atomic potential based models. These potential models have previously been shown to give a good description of the bulk structure of the MgSiO$_3$ perovskite (Wright and Price 1993; Wright and Price 1989; Wall and Price 1989; Price et al. 1989; Wall et al. 1986; Watson et al. 2000), and in this study we have investigated whether they can also be used to model perovskite surfaces, by comparing...
the results that they give with the more accurate ab initio simulations.

Our interest in surface structures is driven by the eventual need to understand the rheological properties of the lower mantle, which one presumes depend upon the behaviour of polycrystalline aggregates of MgSiO$_3$-rich perovskite, their grain boundary processes, and possibly on their crystal morphology. Furthermore, because of the reactivity of surfaces with respect to small molecules (e.g. H$_2$O), ionic impurities (such as Ca$^{2+}$), and other trace elements present in the mantle, it is vital to understand which surface structures (e.g. non-polar (less reactive) or polar (reactive) surfaces) are favoured in MgSiO$_3$ perovskite. The work presented here lays the foundations for further modelling of the rheology and bulk properties of perovskite rock, which will critically depend upon grain boundary and related surface processes. In the current investigation we discuss both non-polar surfaces as well as polar surfaces. However, owing to large surface reconstructions, the latter type of surfaces are often less stable than the non-polar surfaces, causing special computational problems. Consequently, less information on these types of surfaces is reported.

In this paper we briefly summarize the salient aspects of the perovskite structure, and critically discuss the ab initio and empirical methods that we use, with particular reference to the problems of modelling surfaces. In section 4, we present the detailed results of our simulation, and end by comparing our predicted equilibrium morphology for MgSiO$_3$ perovskite with that of crystals recently synthesized in a multi-anvil high pressure cell.

Geometrical structure

MgSiO$_3$ perovskite crystallises with an orthorhombic structure—space group $Pbnm$ (Ross and Hazen 1989), with the tetravalent cations (Si) in an octahedral environment (see Fig. 1), forming a linked network of corner sharing SiO$_6$ octahedra, with the divalent cations (Mg) occupying a distorted 12-fold coordinated site. As can be seen from Fig. 1, the SiO$_6$ octahedra tilt while the Mg$^{2+}$-ions are in a lower symmetric site within the SiO$_6$-cages.

There is no experimental data on the surface structures of MgSiO$_3$, so we compare our results with the information that is available on BaTiO$_3$, PbTiO$_3$ and SrTiO$_3$ perovskites. These phases have been well studied because of their importance as ferroelectric materials. The relationships between the crystallographic space groups of the latter structures and the orthorhombic structure of MgSiO$_3$ are well known, but for clarity are highlighted here. The orthorhombic structures of ATiO$_3$ (A = Ba, Pb and Sr) can be derived from their cubic structure, which belongs to space group $Pm$3$m$ (Wykoff 1965; see Fig. 2a), by a distortion first in the $<001>$ and secondly in the $<110>$ directions. The space group of

Fig. 1a View along the $c$-axis and b along the $b$-axis of the $Pbnm$ perovskite structure of MgSiO$_3$. SiO$_6$-octahedra marked in light grey, Mg-ions and O-ions are marked as larger and smaller dark grey balls

Methods

Computational details

In the current study we employed both density-functional theory (DFT) plane wave calculations and static lattice simulations using effective pair potentials to model the surfaces of MgSiO$_3$. The computational details for the two methods are given separately below. The main reason for using the computer-intensive ab initio plane wave calculations was to check the reliability of the inter-atomic potentials employed in the work, and to obtain a control on how truly transferable they are to the study of surfaces and grain boundaries.
All plane wave calculations were undertaken with the CASTEP code (Payne et al. 1992; Lindan 1999), employing the non-local ultrasoft pseudo-potentials available in the program (Payne et al. 1992; Vanderbilt 1990). The k-point grids employed are $6 \times 6 \times 6$ and $6 \times 6 \times 1$ for the bulk and surface calculations, respectively, and as demonstrated in Fig. 3 a k-point grid of $6 \times 6 \times 6$ is well converged for all the bulk structures investigated. The kinetic energy cut-off ($E_{\text{cut}}$) employed in all calculations was 380 eV. Lattice energies calculated with an $E_{\text{cut}}$ of 380 eV resulted in an error of $\sim 0.01$ eV, compared to the higher $E_{\text{cut}}$ value of 550 eV. In all calculations, we employed the Perdew-Wang91 (Perdew et al. 1992) functional described within the generalised-gradient approximation (GGA). In addition, the density-mixing method, as implemented in CASTEP, was applied.

### Pair-potential methods

The atomistic calculations on the bulk structure were undertaken with the GULP program (Gale 1997; Gale 1996), while for the surface calculations we employed MARVIN’S program (Gay et al. 1999; Gay and Rohl 1995). In both GULP and MARVIN’S program the effective potentials describing the inter-atomic forces are represented by ionic, pair-wise potentials of the form:

$$U_{ij}(r_{ij}) = \frac{Z_i Z_j}{r_{ij}} + \sum_{ij} \varphi_{ij}$$

where the first term represents the Coulombic (long-range) interactions between each pair of ions in the crystal, and the second term describes the short-range interactions in the Buckingham potential:

$$\varphi_{ij}(r_{ij}) = b_{ij} \exp\left(-\frac{r_{ij}}{\rho_{ij}}\right) - \frac{C_{ij}}{r_{ij}^6}$$

For MgSiO$_3$, we employed two sets of inter-atomic potentials: the first potential (denoted P1; see Table 2)
was originally employed by Wall et al. and Wright et al. (Wright and Price 1993; Wright and Price 1989; Wall and Price 1989; Price et al. 1989; Wall et al. 1986; Watson et al. 2000) to model the bulk structure of MgSiO$_3$. In P1 formal charges (i.e. Si=+4, Mg=+2 and O=-2) are used in combination with a shell model on oxygen to simulate the polarisability of the oxygen ions. In addition, this model uses a three-body potential to induce directionality on the bonding around Si.

The second potential set (P2) used is a rigid ion one, with partial ionic charges (see Table 2). This potential was newly derived, and constrained the charge ratio between Mg and Si to be fixed to 1:2, so that this potential could be used to calculate neutral surfaces. The potential was derived employing the same technique as detailed by Oganov et al. (2000). The oxygen-oxygen potential was taken from Gavezzoti (1994), while the $Z_{Si,O_{ref}}$, $Z_{Mg,O_{ref}}$ and $b_{Mg,O}$ were fitted to the experimental crystal structure at 1 atm and 300 K (Ross and Hazen 1989) and to the full elastic constant tensor (Yeganeh-Haeri 1994), using the GULP code ($Z_{O}$ follows from $Z_{O} = -1/3(Z_{Mg} + Z_{Si})$). The $\rho_{ij}$ values are fixed at the values calculated from the first ionisation potential ($I_i$) of atoms using the formula (Urusov 1975):

$$\rho_{ij} = \frac{1.85}{\sqrt{I_i} + \sqrt{I_j}}$$

The success that the P1 and P2 potentials have in reproducing the experimental and DFT data for the crystal structure of MgSiO$_3$ perovskite can be inferred from Table 3.

When simulating the MgSiO$_3$ structure, we need to reproduce the distortion and tilting of the SiO$_6$-octahedras as accurately as possible. In Table 4 the optimised Si-O distances are listed for the different theoretical models, and presented in comparison with the experimentally derived values. It is encouraging to see that the inter-atomic models employed for the MgSiO$_3$ structure reproduce the distortion of the SiO$_6$-octahedras satisfactorily. Likewise the DFT calculations also reproduce well the observed distortion of the SiO$_6$-octahedra.

### Table 2 Inter-atomic potentials employed in the current work

<table>
<thead>
<tr>
<th>Species</th>
<th>Charge</th>
<th>$ij$</th>
<th>$A_{ij}$ (eV)</th>
<th>$r_{ij}$ (Å)</th>
<th>$C_{ij}$ (eV Å$^6$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mg$^{ef}_{ref}$</td>
<td>+2</td>
<td>Mg-O$_{shell}$</td>
<td>1233.8032</td>
<td>0.29453</td>
<td>0.0</td>
</tr>
<tr>
<td>Si$^{ef}_{ref}$</td>
<td>+4</td>
<td>Si-O$_{shell}$</td>
<td>1383.735</td>
<td>0.32082</td>
<td>10.66158</td>
</tr>
<tr>
<td>O$_{shell}^{ref}$</td>
<td>-2.848</td>
<td>O$<em>{shell}$-O$</em>{shell}$</td>
<td>22764.0</td>
<td>0.1490</td>
<td>27.88</td>
</tr>
<tr>
<td>O$_{core}^{ref}$</td>
<td>+0.848</td>
<td></td>
<td></td>
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<td></td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>Core-shell spring constants</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k = 74.92$ eV Å$^2$ in MgSiO$_3$</td>
</tr>
<tr>
<td>O$<em>{shell}$-Si-O$</em>{shell}$ three body spring constant $k = 2.09724$ eV rad$^{-1}$, short range cut off 12 Å</td>
</tr>
</tbody>
</table>

### Table 3 Optimised structures and lattice energies of the GGA and inter-atomic potential calculations of MgSiO$_3$ perovskite, compared with experimental data and previous ab initio calculations.

<table>
<thead>
<tr>
<th>Property</th>
<th>GGA*</th>
<th>P1*</th>
<th>P2*</th>
<th>LDA*</th>
<th>GGA*</th>
<th>Exp.</th>
</tr>
</thead>
<tbody>
<tr>
<td>a (Å)</td>
<td>4.795</td>
<td>4.872</td>
<td>4.784</td>
<td>4.789</td>
<td>4.834</td>
<td>4.775$^b$</td>
</tr>
<tr>
<td>b (Å)</td>
<td>4.931</td>
<td>4.963</td>
<td>4.903</td>
<td>4.922</td>
<td>4.983</td>
<td>4.932$^b$</td>
</tr>
<tr>
<td>$E_f$ (kJ/mol)</td>
<td>25.8</td>
<td>-45.7</td>
<td>-301.0</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Bulk moduli (GPa)</td>
<td>–</td>
<td>322.88</td>
<td>281.97</td>
<td>257.8</td>
<td>231.3</td>
<td>264.0$^c$</td>
</tr>
</tbody>
</table>

Values in *italics* denote temperature corrected lattice parameters and bulk moduli reported by Oganov et al. (2001), and star indicates current work

$^a$ Oganov et al. (2001)

$^b$ Ross and Hazen (1989)

$^c$ Yeganeh-Haeri et al. (1994)
structure can be cleaved in several different planes, representing different MgO- and SiO₂-terminations, e.g. for the {011} surface we constructed one type of MgO- and two types of SiO₂-terminated surfaces, and for the {111} and {101} surfaces we created two MgO- and SiO₂-types of surfaces. Consequently, the surface terminations of the different surfaces have to be carefully investigated. In Table 4 we show examples of different surface terminations examined here.

A particular problem is caused by the polar {111}, {101} and {011} surfaces, which are usually less stable than the non-polar surfaces, and are known to undergo complicated surface reconstructions (Heinrich and Cox 1996). There are many possible surface structures of these polar surfaces, and because of their complexity, only the non-reconstructed surfaces were considered in the current study, i.e. those surfaces that can be cut out from the geometry optimised bulk structure.

**Static lattice simulations**

In MARVIN’S program, the simulated slabs are periodic in two dimensions (x and y) with a finite thickness in the third z-direction, corresponding to the direction perpendicular to the surface. Furthermore, the slab is divided into two regions, a region in which all the atomic coordinates are fully relaxed, and a second region in which the atomic coordinates are kept fixed to the optimised bulk values. These are denoted as Region I and II, respectively. In all calculations, the size of Region I was carefully investigated to make sure that the atomic displacements are completely relaxed and that surface energies were converged with respect to regions of larger thickness. We found that for a slab thickness of 16 surface layers for the {001} surfaces (see Figs. 4a and b), the surface energies and optimised distances are well converged. However, in all calculations we employed slab thicknesses of Regions I and II corresponding to 28 and 56 surface layers, respectively. For all calculations employing MARVIN’S program we used a 2x2 supercell, to allow for the possibility of commensurate surface reconstructions. We found, however, that surface energies only differ by ca. 0.002 J/m² between the 2x2 and 1x1 supercells of the {001} surfaces, suggesting that no surface reconstructions are observed in the computational cell.

**Plane wave methods**

The periodic boundary conditions employed by CA-STEP require the surfaces to be modelled as repeated slabs, separated by a ‘pseudo-vacuum’. In the current study a pseudo-vacuum of 10 Å was applied. The {001} terminated slabs are Type 1 surfaces (Tasker 1979), but due to the low symmetry, a dipole moment exists perpendicular to the surfaces (see Fig. 5). This makes the SCF cycles very difficult to converge, and to avoid this problem the slabs were constructed to have a central plane of symmetry, that therefore gave two equivalent surfaces (Fig. 5c). As for the atomistic simulations, we optimised the thickness of the slab to make sure that surface energies and atomic displacements were fully converged. We found that surface energies are well converged for the {001} surface (see Fig. 6), employing a slab thickness of 17 surface layers.

**Table 4** Optimised tilting and distortion of SiO₆-octahedra for MgSiO₃-perovskite employing the inter-atomic potentials described in the text in comparison to the experimental data and GGA calculations. Notations of the bond-distances (Si-O) and tilt angles φ are given in Fig. 1b.

<table>
<thead>
<tr>
<th></th>
<th>P1</th>
<th>P2</th>
<th>GGA</th>
<th>Exp.*</th>
</tr>
</thead>
<tbody>
<tr>
<td>Si-OI</td>
<td>1.830</td>
<td>1.793</td>
<td>1.792</td>
<td>1.802</td>
</tr>
<tr>
<td>Si-OII</td>
<td>1.818</td>
<td>1.778</td>
<td>1.791</td>
<td>1.791</td>
</tr>
<tr>
<td>Si-OIII</td>
<td>1.814</td>
<td>1.775</td>
<td>1.777</td>
<td>1.789</td>
</tr>
<tr>
<td>φ</td>
<td>145.5</td>
<td>148.5</td>
<td>148.6</td>
<td>146.2</td>
</tr>
</tbody>
</table>

* Ross and Hazen (1989)
Surface energies

The fact that we are required to use slabs with two equivalent surfaces in our plane wave calculations results in having surfaces that have different numbers of MgO and SiO$_2$ formula units in the surface slabs and in the bulk unit cell. One must therefore be careful when calculating the energies of such MgO- and SiO$_2$-terminated surfaces. In this paper, we quote surface energies obtained from the average energy for the two types of termination, thus:

$$E_{\text{surf}} = \frac{1}{4} \left( E_{\text{MgO slab}} + E_{\text{SiO}_2 \text{slab}} - nE_{\text{bulk}} \right) \quad \text{Surface Area}$$

where the factor of 1/4 comes from the fact that we create four surfaces in the cleavage procedure (see Fig. 5c), and the factor $n$ corresponds to the number of formula units of MgSiO$_3$ in the bulk. $E_{\text{bulk}}$ is the total energy of the primitive unit cell of the perovskite structure calculated under the same conditions as the slabs.

Obtaining surface energies as a function of surface termination in MARVIN’S program is more straightforward, as they are obtained from:

$$E_{\text{surf}} = \frac{E_{\text{slab}} - nE_{\text{bulk}}}{\text{Surface Area}}$$

Comparison between lattice static and plane wave surface calculations

To investigate the transferability of the pair-potentials from bulk to surface calculations we started by comparing the average surface energies of the MgO and SiO$_2$-terminated {001} surfaces (employing equation 2) calculated using the inter-atomic potential models (P1 and P2) with the value obtained using our high level ab initio calculations (employing equation 1). In Fig. 6 we present average surface energies for the MgO and SiO$_2$-terminated {001} surfaces obtained within the GGA Hamiltonian as well as the inter-atomic potential based P1 and P2 models. It can be seen that the P1 model overestimates the surface energies compared with the DFT calculations by about 30% (0.5 eV). In contrast surface energies calculated with our P2 model are only overestimated by ca. 10% (0.2 eV) relative to the surface energies obtained from our GGA calculations.

In general, surface energies increase with decreasing ion size and increasing ionic charge. An example of this behaviour is demonstrated by Kotomin et al. (2001). In their study (on SrTiO$_3$ thin films) it is reported that the different DFT Hamiltonians all result in effective charges and surface energies in good agreement to each other. In contrast HF calculated charges are (as expected) higher than those reported for the DFT methods, and the reported surface energies are correspondingly overestimated compared with those obtained with the DFT methods. The higher surface energies calculated with the P1 inter-atomic potential set are therefore probably related to the use of formal charges in this model.

Another important contribution to the surface energy is made by the geometric relaxations of the surfaces when they are created. Here we calculate the relaxation energies of the {001} surfaces for the P1, P2 and GGA models of MgSiO$_3$. Surface energies reported are the average surface energy of the MgO- and SiO$_2$-terminated surfaces, calculated according to equation 1 and 2 for the CASTEP and MARVIN’S calculations, respectively. Energies given in J/m$^2$. 

![Fig. 5a](image_url) Schematic picture of a Type 1 surface according to Tasker (1979). Q denotes total charge of surface layer, and μ the dipole moment. b [001] slab containing both an MgO and SiO$_2$-terminated surface. c The upper figure shows a slab containing two MgO-terminated surfaces, and the lower figure a slab with two SiO$_2$-terminated surfaces. Both slabs contain a mirror plane, indicated as dashed lines. Numbering of surface layers is given

![Fig. 6](image_url) Surface energies for the P1, P2 and GGA models of MgSiO$_3$. Surface energies reported are the average surface energy of the MgO- and SiO$_2$-terminated surfaces, calculated according to equation 1 and 2 for the CASTEP and MARVIN’S calculations, respectively. Energies given in J/m$^2$.
affect the total calculated surface energies in a less predictable way.

To establish the complete behaviour of the interatomic potential models we also report surface displacements perpendicular to the surface plane for the \{001\} surfaces (see Fig. 7). Analysing the displacements of the geometry optimised surfaces relative to the non-geometry optimised structures, we find that the P1 model reproduces the same behaviour as the GGA calculations, while the P2-calculations fail to reproduce the rumpling of both the MgO- and SiO\textsubscript{2}-terminated surfaces. Potential set P2 predicts an outward relaxation for only one type of the O-ions, while both the GGA and P1 calculations show that the two types of O-ions are outwardly relaxed, while the Si-ions are inwardly relaxed on the SiO\textsubscript{2}-terminated surface (see Fig. 4b). For the MgO-terminated surface (see Fig. 4a) both the O and Mg ions are inwardly relaxed with similar displacements. Although the surface displacements observed for the P1 potential are smaller than the surface relaxations predicted with our GGA calculations, the P1 potential gives the same relative directions for both the MgO- and SiO\textsubscript{2}-terminated surfaces as the DFT calculations, and the rumpling of the surfaces is, therefore, similar. The source of the failure of the surface displacements with the P2 model is likely to originate from the rigid ion oxygen potential used in this model, which fails to reproduce the polarisability of the surface oxygens.

To conclude, we find that it is clearly important to use a shell-model to correctly reproduce the polarisation of the oxygen ions at the surface structure, and all atomistic simulations in the results section are undertaken with the P1 potential except in Section 4.1 were we make further comparisons between the P1 and P2 potentials.

### Results

### Surface stabilities

We commenced our investigation by performing a number of inter-atomic potential calculations on low index non-polar surfaces (see Fig. 4), including various surface terminations, denoted MgO- or SiO\textsubscript{2}-terminated depending on the ionic species dominating the surface layer. As can be seen from Fig. 8 the most stable surface, employing the P1 model, is the MgO-terminated \{001\} surface. The next most stable surface is the MgO-terminated \{110\} surface, which has a surface energy that is only 0.3 J/m\textsuperscript{2} higher than that of the \{001\} surface—a result that follows from the similarity in the structures of the two surfaces.

The polar \{111\}, \{101\} and \{011\} surfaces of perovskites are known to reconstruct to sustain charge...
neutral. These reconstructions are often complicated and depend on the environment, such as oxidising or reducing nature of the atmosphere. However, here we present a number of different unreconstructed surface structures, and we find that these polar surfaces also favour an MgO-termination, with surface structures similar to those shown in the upper part of Fig. 4. Indeed for the P1 potential, all the surfaces we investigated showed that the MgO-termination is more favourable than the SiO2-terminated surfaces. We also find that the surface energies (see Fig. 8) for the different SiO2-terminated {101} and {111} surfaces are similar, which is related to the similar surface structures after relaxation. We have no experimental data with which to compare our optimised polar surfaces of MgSiO3, and it is, therefore, difficult to resolve the true structures of these surfaces: i.e. to establish whether they undergo large surface reconstructions or remain essentially unreconstructed.

For comparison we also show the results obtained for model P2 in Fig. 8, and it can be seen that in fact that both potentials P1 and P2 predict the same relative order of the surface energies. We are, therefore, confident that surface structures and predicted morphologies (discussed in Section 5 below) show the correct general behaviour, even though the absolute surface energies are overestimated with the P1 model compared with those obtained with the GGA and P2 simulations (see Fig. 6). The largest discrepancy between the P1 and P2 potential is demonstrated for the {001} and {110} surfaces, which can be associated with the different surface relaxations of the P2 model compared to the P1 and GGA calculations discussed above.

### Surface geometries

In the current section we analyse the surface relaxations listed in Table 5. In this table, we report the surface displacements both within the plane of the surface (i.e. in the x- and y-directions) as well as perpendicular to the surfaces (z-direction). In summary, we find that for both the MgO- and SiO2-terminated {001} surfaces the cations (Mg2+ and Si4+) are inwardly relaxed perpendicular to the surface plane, while the O2- ions show smaller displacements often within the surface plane. Consequently, both the SiO2- and the MgO-terminated structures are oxygen terminated. Additionally, it can be seen that the SiO2-terminated surface shows larger relaxations than the MgO-terminated surface. Hence, the latter surface termination causes a smaller overall structural perturbation.

Owing to the similarity of the structures of the {001} and {110} surfaces we expect these surfaces to behave in a comparable way, but as consequence of symmetry the outermost Mg2+ ions on the {110} surface only displace within the x- and z-directions, while on the {001} surface the translation of the Mg2+ ions occur also in the y-direction.

### Comparison with other perovskite systems

Surface energies for different surface terminations of the cubic and tetragonal structures of SrTiO3, BaTiO3 and PbTiO3 are reported in Padilla and Vanderbilt (1998), Padilla and Vanderbilt (1997), Kotomin et al. (2001), Heifets et al. (2001), Heifets et al. (2000) and Meyer et al. (1999). Heifets et al. (2000) report inter-atomic potential calculations on the {100} surfaces of the cubic perovskite SrTiO3, employing MARVIN’S program. In this latter investigation the SrO-terminated {001} surface is reported to be the energetically most stable surface (E = 1.32 J/m²), in agreement with our calculations on MgSiO3 for which the MgO-terminated {001} surface is the energetically most stable surface (E = 2.2 J/m²). However, plane wave calculations by Vanderbilt’s group (Meyer et al. 1999; Padilla and Vanderbilt 1998), formulating the surface energy as a function of the chemical potential of TiO₂, predict that when the TiO₂-terminated
surfaces studied are not unreasonably large, only being Si-terminated surfaces, the energies of all the other polar surfaces are reported to be energetically less sensitive to the different types of surface terminations than the {110} surfaces, i.e. for the {111} surfaces the calculated surface energies are all close to \(\sim 2.5 \text{ J/m}^2\) independent of the surface composition, while for the {110} surface energies vary between 1.1 and 3.1 J/m\(^2\) dependent on surface geometry and stoichiometry. For the polar surfaces of MgSiO\(_3\) we find a similar behaviour: the {111} surface (structural equivalent to the {111} surface of SrTiO\(_3\)) is less structure-sensitive than the {011} surface (comparable with the {110} surface of SrTiO\(_3\)) as shown in Fig. 8. One of the {011} surfaces is Si-terminated, and as such it contains unsaturated bonds. Consequently, it has a higher surface energy (\(E_s \sim 5.5 \text{ J/m}^2\)) than the other surface terminations. We know from silica (Koudriachova et al. 2001; Bart and Gautier 1994; Legrand 1999) and silicon (Nörenberg and Briggs 1999; Çakmak and Srivastava 2003) that these surfaces readily reconstruct or form different types of hydroxylated surfaces (de Leeuw et al. 1999; Litton and Garofalinia 2001; Mazzara et al. 1999; Legrand 1999). With the exception of the Si-terminated surfaces, the energies of all the other polar surfaces studied are not unreasonably large, only being about 1.5 J/m\(^2\) higher than the non-polar surfaces, an observation that agrees with the surface energies calculated for SrTiO\(_3\) and BaTiO\(_3\).

Experimental studies on BaTiO\(_3\) (Lo and Samorjai 1977; Ferrer and Samorjai 1980; Tanaka and Kawai 1996; Sigmund et al. 1997; Sekiguchi et al. 1998; Hagendorf et al. 1998) and SrTiO\(_3\) (Bando et al. 1995; Brunen and Zegenhagen 1997; Erdman et al. 2002) show that under oxidising and reducing conditions complicated surface reconstructions may occur, resulting in both AO\(_{3-x}\)- and TiO\(_x\)-terminated surfaces. However, in a number of the latter investigations unreconstructed polar surfaces are also reported, while other oxides such as MgO only show reconstructed polar {111} surfaces (Heinrich 1976). The reason for this is believed to be that the ATiO\(_3\) perovskites form insulating polar surfaces, while the polar {111} surface of MgO is found to be metallic and as such less stable than the insulated surfaces of ATiO\(_3\) (Pojani et al. 1999b).

### Morphology

It has been suggested (e.g. Karato et al. 1992, 1995) that the dominant mechanism for the deformation of MgSiO\(_3\)-perovskite in the lower mantle will depend on its grain size; i.e. deformation of small grain size aggregates occurs by diffusion or superplastic creep, whereas dislocation creep or twinning will be favoured in larger grain size assemblages. Hence, the rheological properties of the lower mantle are likely to be affected by the grain size and shape of MgSiO\(_3\)-perovskite. Although the grain shape in a dynamically deforming system will be different from that of its equilibrium morphology, it is generally true that at moderate strain rates minerals that develop equant equilibrium morphologies do not develop large shear induced morphological anisotropy. Although the shape of MgSiO\(_3\)-perovskite crystals developed in a convecting mantle cannot yet be established, it is possible to calculate its equilibrium crystal morphology. This crystal morphology is also important for understanding the reactivity of MgSiO\(_3\)-perovskite, i.e. if the exposed crystal habits are polar (reactive) or non-polar (less reactive) surface structures.

In the current investigation we employ three different models to predict the morphology, i.e. the Bravais-Friedel-Donnay-Harker (BFDH) model, the Equilibrium model and the Growth model (Gay and Rohl 1995). In the BFDH model, the morphology is obtained by analysing the crystal structure (cell parameters and space group), assuming that surfaces with large inter-planar distances (\(d_{hkl}\)-values) control the crystal morphology, because the growth rates of these faces are the rate determining growth steps. In the Equilibrium model the morphology is derived from calculated surface energies and their orientation dependence to the crystallographic axes. Making the assumption that faces with the lowest surface energies and largest inter-planar distances control the growth rate and that these surfaces determine the crystal morphology. In the Growth model the growth rates of the surface planes are predicted from the attachment energies, defined as the energy gained when a ‘growth slice’ is attached to a face of the growing crystal. The growth rate of a given crystal face is taken to be proportional to the absolute value of the attachment energy, and so faces with the lowest attachment energies will be the slowest growing and have the most morphological importance.

The predicted morphologies obtained by employing these three models are shown in Fig. 9. It can be seen that both the BFDH and Equilibrium models (Figs. 9a and b) predict a faceted morphology, while the Growth model predicts a rectangular prismatic morphology (Fig. 9c). However, analysing the surface areas it is found that for all three models the dominating faces are the {001} and {110} surfaces, which we have shown above have equivalent surface energies owing to the similarity of their surface structures (see Fig. 4). Because the dominating surface structures are the energetically most stable non-polar surfaces (see Fig. 8) MgSiO\(_3\)-perovskite crystals are expected to be less reactive than if the polar surfaces were the dominating surfaces.
Fig. 10 shows a scanning electron microscopy (SEM) image of flux-grown MgSiO$_3$ perovskite crystals. These crystals were grown from a starting synthetic perovskite of composition 0.997(MgSiO$_3$)$_{0.003}$(NaSiO$_2$.5), which was loaded in a NaCl pressure medium and recrystallised at 24 GPa and 1950°C for 30 min. NaCl is a liquid under these experimental conditions and idiomorphic perovskite crystals grew by a solution-precipitation process. The resultant MgSiO$_3$ perovskite shows a dominant prismatic to cubic habit with minor octahedral facet development. This can be produced by development of {001}, {010} and {100} pinacoids with {111} form, or {001} pinacoid with {110}, {101} and {011} forms of the pseudocubic setting in orthorhombic (Pbnm) symmetry, the latter being consistent with Fig. 9e. The flux-grown MgSiO$_3$ crystals in general exhibit a particle size of ca. 200 μm. Previous observations suggest that the morphologies predicted from the Growth model are often found to reproduce best the experimental observations considering larger crystal and geological species, while the Equilibrium model morphology often agrees better with experimentally observed perovskite morphologies. More work is needed on the theory of crystal growth before the appropriateness of the competing models can be effectively established.

**Summary**

This paper is the first in a series in which we aim to investigate the surface structures and grain boundaries of MgSiO$_3$ perovskite. Surface simulations are computationally demanding, and so one would wish to do them with computationally efficient simulations, but one needs also to establish the accuracy of the simulation. In this study we have confirmed that simulations of perovskite surfaces can be done using effective interatomic potential calculations, and that these simulations predict the same behaviour as more computationally intensive high level ab initio methods. Our calculations show that it is important to employ a shell-model potential to correctly reproduce the surface structures. However, to reproduce surface energies, in agreement
with GGA calculations, the potential should be derived with partial charges on the ionic species.

Our atomistic simulations predict the MgO-terminated \{001\} surface to be the energetically most stable surface for MgSiO$_3$ perovskite. As expected, the SiO$_2$-terminated surfaces are less stable than the MgO-terminated surfaces, because the SiO$_2$-terminated surfaces contain unsaturated Si-bonds. As a result of surface relaxations, all the surfaces we investigated are oxygen terminated. For the polar \{111\}, \{101\} and \{110\} surfaces we also find that the ionic displacements result in more dense surface structures than observed for the unrelaxed surfaces.

In the future we intend to investigate the grain boundary structures of MgSiO$_3$ perovskite. For this we need to know the most probable surfaces exhibited by MgSiO$_3$ perovskite, and so we employed three different models to predict its most likely morphology. All three models showed that the largest proportion of the perovskite surface is likely to be dominated by the \{001\} and \{110\} faces. However, dependent on the model we use, we predict a facetted or prismatic morphology for this mineral. SEM pictures of the analogue structure CaTiO$_3$ show facetted crystal particles in good agreement with our BFDH and equilibrium models, while the Growth model shows better agreement with flux grown crystals of MgSiO$_3$ perovskite. Because the crystal morphology is environmentally dependent, we conclude that our predicted crystal morphologies are plausible, and we await more experimental studies on perovskite morphology before we can differentiate between the varieties of competing morphology models currently available.

Acknowledgements We wish to thank S. Jacobsen and F. Heidelbach for assistance in collecting the SEM image of the flux-grown Mg$_2$SiO$_4$ crystal, and D. S. Coombes, F. Corà and K.W. Wright for valuable discussions. Financial support from NERC is gratefully acknowledged (grant No(s): NER/T/S/2001/00855; NER/O/S/2001/01227 and GR9/02072). We are also thankful to EPSRC and NERC for the computer facilities provided at the Royal Institution of Great Britain, University College London, Bath University and the High Performance Computing facilities of the University of Manchester. DPD is grateful for fellowships from the Alexander von Humboldt Stiftung and the Royal Society.

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